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Nonclassicality and Decoherence of Photon-added Thermo-invariant Coherent State

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Abstract By introducing a kind of new quantum state—Photon-added thermo invariant coherent state (PATCS), we discuss its nonclassicality in terms of the negativity of Wigner function (WF) after deriving its analytical expression. It is found that the Wigner function is related to Lagurre-Gaussian function. We then study the effect of decoherence (a thermal environment) on the PATCS according to its WF (also related to Lagurre-Gaussian function). It is shown that it is not possible for WF to present the negative region when the decay time $\kappa t > \frac{1}{2}\ln\frac{2\bar{n}+2}{2\bar{n}+1}$.

Keywords Photon-added pair coherent state · Wigner function · Nonclassicality

1 Introduction

Recently, the investigation of non-Gaussian quantum states prepared by photon addition and subtraction manipulation is full of interest [1–12]. The reasons are that subtracting or adding photons from/to a Gaussian field are plausible way to conditionally manipulate nonclassical state of optical field. In fact, such methods allowed the preparation and analysis of several states with negative Wigner functions, such as one- and two-photon Fock states [13–16], delocalized single photons [17, 18], single-photon-added coherent states [1, 2] and thermal states [3], single-photon-subtracted squeezed states [4, 5] and so on, which have been realized in experiments. In addition, photon subtraction or addition is also applied to improvement of the entanglement between Gaussian states [14], loophole-free tests of Bell's inequality [19], and quantum computing [20].

On the other hand, the nonclassicality of quantum states can be characterized by some nonclassical properties such as negativity of the Wigner function (WF), squeezing in one

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of the phasequadrature components, and sub-Poissonian photon statistics. Among them, the partial negativity of the WF is indeed a good indication of the highly nonclassical character of the state [21, 22]. Therefore it is worth of obtaining the WF for any states and to measure their negative values. In addition, the negativity of WF is also used to describe the decoherence of quantum states, e.g., the excited coherent state in both photon-loss and thermal channels [7, 23], the single-photon subtracted squeezed vacuum state in both amplitude decay and phase damping channels [4, 5], and so on. In [24] the WF of thermo invariant coherent state is derived by using the thermal Wigner operator of thermo-field dynamics in the coherent thermal state representation and the IWOP technique [25, 26]. For other examples of calculating the WFs, we refer to [27–30].

In this paper, we shall introduce a kind of new quantum state—Photon-added thermo invariant coherent state (PATICS), and discuss its nonclassicality in terms of the negativity of Wigner function after deriving its analytical expression. Then we investigate its decoherence in thermal environments according to the partial negativity of WF. Our work is arranged as follows. In Sect. 2, we briefly review the thermo invariant coherent state, and then introduce the Photon-added thermo invariant coherent state. By using the coherent state representation of Wigner operator, in Sect. 3, we derive the analytical expression of WF for PATICS, which is related to Lagurre-Gaussian function. It is shown that when the photon-added number (m) is taken different values, the PATICS exhibits different nonclassicality. Moreover, the nonclassicality is more evident when $m+q$ (q defined in (1) below) is an odd number for the PATICS. Section 4 is devoted to discussing decoherence of the PATICS in thermal environments based on the negativity of the WF. It is found that it is not possible for WF to present the negative region when the decay time $\kappa t > \frac{1}{2}\ln\frac{2\bar{n}+2}{2\bar{n}+1}$ (see (28) below), where $\bar{n}$ denotes the average thermal photon number in the environment with dissipative coefficient κ. Conclusions are involved in the last section.

2 Brief Review of Thermo Invariant Coherent State

In [31], by analysing the characters of thermo field dynamics, the thermo invariant coherent state $|\zeta, q\rangle$ has been introduced in thermal equilibrium, whose expression in two-mode Fock space (consisted of one real mode and one tidle mode) is

$$|\zeta, q\rangle = C_q \sum_{n=0}^{\infty} \frac{\zeta^n}{\sqrt{(n+q)!n!}} |n+q, \tilde{n}\rangle, \tag{1}$$

where q and ζ are an integer and complex, respectively; $|\tilde{n}\rangle$ is the fictitious mode Fack state, as well as C_q is the real normalization constant,

$$C_q = \left[\sum_{n=0}^{\infty} \frac{|\zeta|^{2n}}{n!(n+q)!}\right]^{-1/2} = \left[(i|\zeta|)^{-q} J_q(2i|\zeta|)\right]^{-1/2}, \tag{2}$$

in which J_q is the Bessel function

$$J_q(x) = \sum_{n=0}^{\infty} \frac{(-1)^q}{n!(n+q)!} \left(\frac{x}{2}\right)^{2n+q}, \tag{3}$$

whose generation function is $e^{ix\sin t} = \sum_{n=-\infty}^{\infty} J_n(x) e^{int}$. The state $|\zeta, q\rangle$ is the common eigenvector of the pair annihilation operator $a\tilde{a}$ and the "total" energy operator $(a^\dagger a - \tilde{a}^\dagger \tilde{a})$, i.e.,

$$a\tilde{a}|\zeta,q\rangle=\zeta|\zeta,q\rangle,\qquad (a^{\dagger}a-\tilde{a}^{\dagger}\tilde{a})|\zeta,q\rangle=q|\zeta,q\rangle, \tag{4}$$

where $[a,a^{\dagger}]=[\tilde{a},\tilde{a}^{\dagger}]=1$ and $[a,\tilde{a}^{\dagger}]=[\tilde{a},a^{\dagger}]=[a,\tilde{a}]=[\tilde{a}^{\dagger},a^{\dagger}]=0$, i.e., the operators in fictitious space are commutative with those in real space. From (4) we can see clearly the physical meaning of $|\zeta,q\rangle$ is: annihilating a quantum of the system and meanwhile annihilating a hole with negative energy in the reservoir will not change the energy of the whole system in thermal equilibrium.

Next, we introduce the poton-added thermo invariant coherent state (PATICS). Theoretically the PATICS can be generated by repeatedly operating the photon creation operator $a^{\dagger}$ on $|\zeta,q\rangle$,

$$|\zeta,q,m\rangle\equiv C_{q,m}a^{\dagger m}|\zeta,q\rangle, \tag{5}$$

where $C_{q,m}$ is the real normalization factor, which can be determined by $1=C_{q,m}^{2}\langle\zeta,q|a^{m}a^{\dagger m}|\zeta,q\rangle$.

Using the expression $|\zeta,q\rangle$ it is easy to see that

$$\begin{aligned}|\zeta,q,m\rangle&=C_{q,m}C_{q}\sum_{n=0}^{\infty}\frac{\zeta^{n}a^{\dagger m}}{\sqrt{(n+q)!n!}}|n+q,\tilde{n}\rangle\\&=C_{q,m}C_{q}\sum_{n=0}^{\infty}\frac{\zeta^{n}\sqrt{(n+m+q)!}}{(n+q)!\sqrt{n!}}|m+n+q,\tilde{n}\rangle,\end{aligned} \tag{6}$$

which leads to

$$1=\langle\zeta,q,m|\zeta,q,m\rangle=C_{q,m}^{2}C_{q}^{2}\sum_{n=0}^{\infty}\frac{|\zeta|^{2n}(n+m+q)!}{[(n+q)!]^{2}n!}, \tag{7}$$

thus

$$C_{q,m}^{2}C_{q}^{2}=\left\{\sum_{n=0}^{\infty}\frac{(n+m+q)!|\zeta|^{2n}}{[(n+q)!]^{2}n!}\right\}^{-1}. \tag{8}$$

It is evident that $C_{q,0}^{2}=1$ when $m=0$.

3 Negativity of the PATICS

The Wigner function is a useful measure for studying the nonclassical features of quantum states. For single-mode quantum state, the Wigner operator $\Delta(\alpha,\alpha^{*})$ is defined by [32–34]

$$\begin{aligned}\Delta(\alpha,\alpha^{*})&=e^{2|\alpha|^{2}}\int\frac{d^{2}z}{\pi^{2}}|z\rangle\langle -z|e^{-2(z\alpha^{*}-z^{*}\alpha)}\\&=\frac{1}{\pi}\colon\exp\left[-2(a^{\dagger}-\alpha^{*})(a-\alpha)\right]\colon,\end{aligned} \tag{9}$$

where : : denotes the normal ordering form of operators, $\alpha=(x+ip)/\sqrt{2}$ and WF of quantum state ρ can be calculated as $W(\alpha,\alpha^{*})=\mathrm{tr}[\rho\Delta(\alpha,\alpha^{*})]$.

According to (5) we know that the density operator of PATICS is $\rho = \mathrm{tr}_{\tilde{a}}[|\zeta, q, m\rangle\langle\zeta, q, m|]$, where $\mathrm{tr}_{\tilde{a}}$ denotes the tracing for tidle mode $\tilde{a}$. Thus the WF of PATICS is given by

$$\begin{aligned} W(\alpha,\alpha^*) &= \mathrm{tr}\big[\mathrm{tr}_{\tilde{a}}[|\zeta,q,m\rangle\langle\zeta,q,m|]\Delta(\alpha,\alpha^*)\big] \\ &= \langle\zeta,q,m|\Delta(\alpha,\alpha^*)|\zeta,q,m\rangle. \end{aligned} \tag{10}$$

In order to further derive (10), we first notice that the matrix element of Wigner operator $\Delta(\alpha,\alpha^*)$ between number states $\langle m|$ and $|n\rangle$ (see Appendix) i.e.,

$$\langle m|\Delta(\alpha,\alpha^*)|n\rangle = \frac{e^{-2|\alpha|^2}}{\pi\sqrt{m!n!}} H_{m,n}(2\alpha, 2\alpha^*), \tag{11}$$

where $H_{m,n}(\xi,\xi^*)$ is two-variable Hermite polynomials, whose generating function is

$$\sum_{m,n=0}^{\infty} \frac{t^m t'^n}{m!n!} H_{m,n}(\xi,\xi^*) = \exp[-tt' + \xi t + \xi^* t']. \tag{12}$$

Then substituting the Fock representation of PATICS in (5) into (10) and using (11), we have

$$\begin{aligned} W(\alpha,\alpha^*) &= C_{q,m}^2 C_q^2 \sum_{l,n=0}^{\infty} \frac{\zeta^{*l}\zeta^n\sqrt{(l+m+q)!(n+m+q)!}}{(l+q)!\sqrt{l!}(n+q)!\sqrt{n!}} \\ &\quad \times \langle m+l+q, \tilde{l}|\Delta(\alpha,\alpha^*)|m+n+q, \tilde{n}\rangle \\ &= C_{q,m}^2 C_q^2 \sum_{l,n=0}^{\infty} \frac{\zeta^{*l}\zeta^n\sqrt{(l+m+q)!(n+m+q)!}}{(l+q)!\sqrt{l!}(n+q)!\sqrt{n!}} \\ &\quad \times \langle m+l+q|\Delta(\alpha,\alpha^*)|m+n+q\rangle\delta_{l,n} \\ &= \frac{C_{q,m}^2 C_q^2}{\pi} \sum_{n=0}^{\infty} \frac{|\zeta|^{2n} e^{-2|\alpha|^2}}{n![(n+q)!]^2} H_{m+n+q,m+n+q}(2\alpha, 2\alpha^*) \\ &= \frac{C_{q,m}^2 C_q^2}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^{m+n+q}|\zeta|^{2n}(m+n+q)!}{n![(n+q)!]^2} e^{-2|\alpha|^2} L_{m+n+q}\big(4|\alpha|^2\big), \end{aligned} \tag{13}$$

where we have used $\langle\tilde{l}|\tilde{n}\rangle = \delta_{l,n}$ and in the last step, we have used the relation between two-variable Hermite polynomial and Laguerre polynomial,

$$\frac{(-1)^m}{m!} H_{m,m}(x,y) = L_m(xy), \tag{14}$$

where Laguerre polynomial is defined as

$$L_n(x) = \sum_{l=0}^{n} \frac{n!}{(n-l)!l!l!}(-x)^l. \tag{15}$$

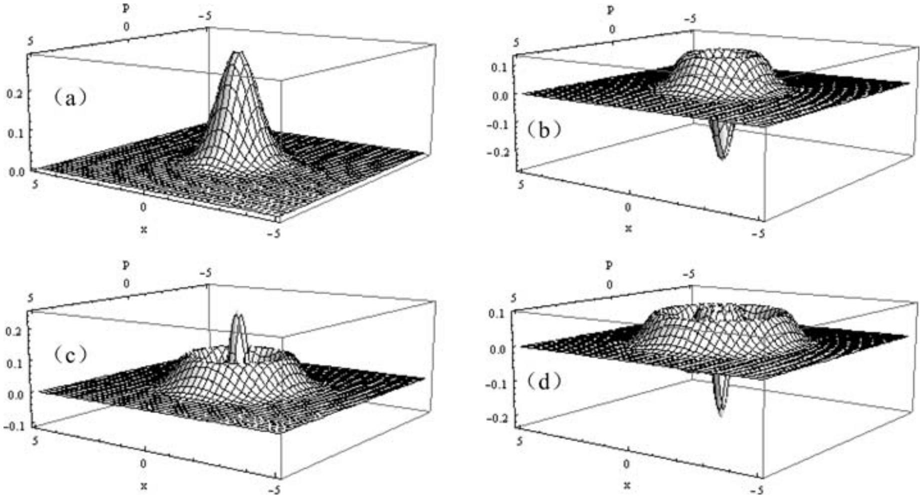

Fig. 1 Wigner function of PATICS for different values of m ($q=0$) (**a**) $m=0$, (**b**) $m=1$, (**c**) $m=2$, (**d**) $m=3$

Equation (13) is just the Wigner function of PATICS. In particular when $m=0$, (13) just reduces to

$$\begin{aligned} W(\alpha,\alpha^*) &= \frac{C_q^2}{\pi}\sum_{n=0}^{\infty}\frac{|\zeta|^{2n}e^{-2|\alpha|^2}}{n![(n+q)!]^2}H_{n+q,n+q}(2\alpha,2\alpha^*) \\ &= \frac{C_q^2}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^{n+q}|\zeta|^{2n}e^{-2|\alpha|^2}}{n!(n+q)!}L_{n+q}\left(4|\alpha|^2\right), \end{aligned} \tag{16}$$

which agrees with the result (32) in [24]. In fact, when $\zeta=q=0$, (16) just becomes the Wigner function of coherent state, $W(\alpha,\alpha^*)=\frac{1}{\pi}e^{-2|\alpha|^2}$.

Next, we shall discuss WF distribution in phase space. Using (13) we plot WF distribution for different photon-added number m and different values q (a given $\zeta=0.2$) in Figs. 1–2. It is easy to see that in the phase space center, there is a downward peak when $m+q$ is an odd number; while for $m+q$ is an even number, there is a upward peak. That is to say, the negativity of WF depend on not only the energy q of the whole system in thermal equilibrium, but also on the photon-added number m. In addition, we can also see that for given q the "space frequency" of distribution function increases with the increasement of number m.

4 Decoherence of PATICS

In this section, we discuss the decoherence of PATICS in a thermal environment according to the evolution of WF. When the m-PATICS evolves in the thermal channel, the evolution of the density matrix can be described by [35]

$$\frac{d\rho}{dt}=\kappa(\bar{n}+1)(2a\rho a^{\dagger}-a^{\dagger}a\rho-\rho a^{\dagger}a)+\kappa\bar{n}(2a^{\dagger}\rho a-aa^{\dagger}\rho-\rho aa^{\dagger}), \tag{17}$$

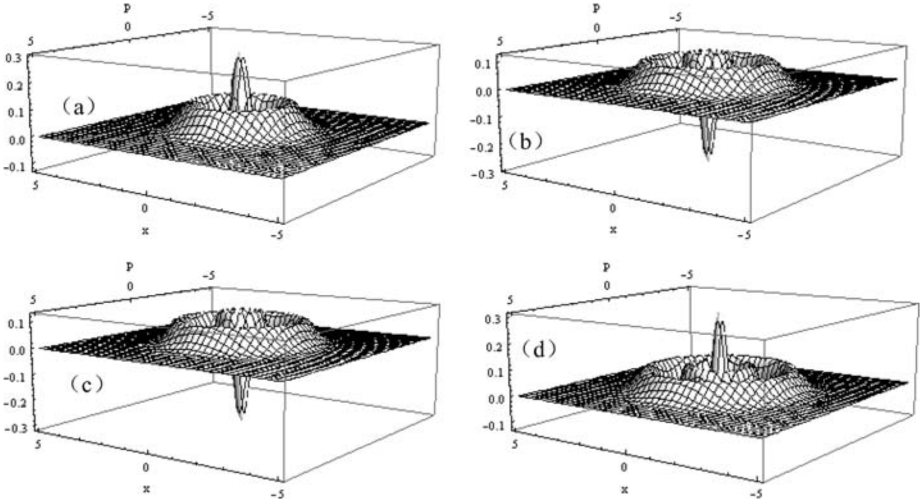

Fig. 2 Wigner function of PATICS for different values of m and q (**a**) $m=1$, $q=1$, (**b**) $m=2$, $q=1$, (**c**) $m=1$, $q=2$, (**d**) $m=2$, $q=2$

where κ represents the dissipative coefficient and $\bar{n}$ denotes the average thermal photon number of the environment. When $\bar{n}=0$, (17) reduces to the master equation describing the photon-loss channel [36, 37]. In [38] Hu and Fan have derived the evolution formula of WF in laser process by using entangled state representation, the case in (17) as its a special example, i.e.,

$$W(\alpha,\alpha^*,t)=\frac{2}{(2\bar{n}+1)T}\int\frac{d^2\beta}{\pi}W(\beta,\beta^*,0)e^{-2\frac{|\alpha-\beta e^{-\kappa t}|^2}{(2\bar{n}+1)T}},\tag{18}$$

where $T=1-e^{-2\kappa t}$ and $W(\beta,\beta^*,0)$ is the Wigner function of the initial state.

Substituting (13) into (18) we have

$$W(\alpha,\alpha^*,t)=\frac{2C_{q,m}^2C_q^2}{\pi(2\bar{n}+1)T}\sum_{n=0}^{\infty}\frac{|\zeta|^{2n}}{n![(n+q)!]^2}G_n(\alpha,\alpha^*,t),\tag{19}$$

where $G_n(\alpha,\alpha^*,t)$ is given by

$$G_n(\alpha,\alpha^*,t)=\int\frac{d^2\beta}{\pi}e^{-2\frac{|\alpha-\beta e^{-\kappa t}|^2}{(2\bar{n}+1)T}}e^{-2|\beta|^2}H_{m+n+q,m+n+q}(2\beta,2\beta^*).\tag{20}$$

Using the generating function of $H_{m,n}(\xi,\xi^*)$ in (12), and the integration formula,

$$\int\frac{d^2z}{\pi}e^{\zeta|z|^2+\xi z+\eta z^*}=-\frac{1}{\zeta}e^{-\frac{\xi\eta}{\zeta}},\quad \mathrm{Re}(\zeta)<0,\tag{21}$$

we can calculate (20) as

$$\begin{aligned} G_n(\alpha,\alpha^*,t) &= e^{-\frac{2|\alpha|^2}{(2\bar{n}+1)T}}\frac{\partial^{m+n+q}}{\partial\tau^{m+n+q}}\frac{\partial^{m+n+q}}{\partial\upsilon^{m+n+q}}e^{-\tau\upsilon}\int\frac{d^2\beta}{\pi} \\ &\quad\times\exp\left\{-\frac{2(2\bar{n}T+1)}{(2\bar{n}+1)T}|\beta|^2+2\left(\frac{\alpha^*e^{-\kappa t}}{(2\bar{n}+1)T}+\tau\right)\beta\right. \\ &\quad\left.+2\left(\frac{\alpha e^{-\kappa t}}{(2\bar{n}+1)T}+\upsilon\right)\beta^*\right\}_{\tau=\upsilon=0} \\ &= \frac{(2\bar{n}+1)T}{2(2\bar{n}T+1)}e^{-\frac{2|\alpha|^2}{2\bar{n}T+1}}\frac{\partial^{m+n+q}}{\partial\tau^{m+n+q}}\frac{\partial^{m+n+q}}{\partial\upsilon^{m+n+q}} \\ &\quad\times\exp\left[-\frac{A}{B}\tau\upsilon+\frac{2\alpha e^{-\kappa t}}{B}\tau+\frac{2\alpha^*e^{-\kappa t}}{B}\upsilon\right]_{\tau=\upsilon=0} \\ &= \frac{(2\bar{n}+1)T}{2(2\bar{n}T+1)}e^{-\frac{2|\alpha|^2}{2\bar{n}T+1}}\left(\frac{A}{B}\right)^{m+n+q}H_{m+n+q,m+n+q}\left[\frac{2e^{-\kappa t}}{\sqrt{AB}}\alpha,\frac{2e^{-\kappa t}}{\sqrt{AB}}\alpha^*\right], \end{aligned} \tag{22}$$

where we have set

$$A=e^{-2\kappa t}-(2\bar{n}+1)T,\qquad B=e^{-2\kappa t}+(2\bar{n}+1)T=2\bar{n}T+1, \tag{23}$$

and used (12) again in the last step in (22).

Further noticing (14) we can reform (22) as

$$G_n(\alpha,\alpha^*,t)=(m+n+q)!\frac{(2\bar{n}+1)T}{2(2\bar{n}T+1)}\left(-\frac{A}{B}\right)^{m+n+q}e^{-\frac{2|\alpha|^2}{2\bar{n}T+1}}L_{m+n+q}\left[\frac{4e^{-2\kappa t}}{AB}|\alpha|^2\right]. \tag{24}$$

Then substituting it into (19) yields

$$\begin{aligned} W(\alpha,\alpha^*,t) &= \frac{C_{q,m}^2C_q^2}{\pi(2\bar{n}T+1)}e^{-\frac{2|\alpha|^2}{2\bar{n}T+1}}\sum_{n=0}^{\infty}\frac{(m+n+q)!}{n![(n+q)!]^2}|\zeta|^{2n}\left(-\frac{A}{B}\right)^{m+n+q} \\ &\quad\times L_{m+n+q}\left[\frac{4e^{-2\kappa t}}{AB}|\alpha|^2\right], \end{aligned} \tag{25}$$

which is just the Wigner function of PATICS in a thermal environment.

In particular, when $t=0$ leading to $T=0$, (25) reduces to

$$W(\alpha,\alpha^*,0)=\frac{C_{q,m}^2C_q^2}{\pi}e^{-2|\alpha|^2}\sum_{n=0}^{\infty}\frac{|\zeta|^{2n}(m+n+q)!}{(-1)^{m+n+q}n![(n+q)!]^2}L_{m+n+q}\left(4|\alpha|^2\right), \tag{26}$$

which corresponds to the Wigner function of PATICS; while for $t\to\infty$ leading to $T\to 1$, and noticing $L_m(0)=1$, thus (25) just becomes

$$W(\alpha,\alpha^*,\infty)=\frac{1}{(2\bar{n}+1)\pi}e^{-\frac{2|\alpha|^2}{2\bar{n}+1}}, \tag{27}$$

i.e., the WF of thermal vacuum state.

In Fig. 3, the WFs of the PATICS with $m=1, q=0$ are plotted in phase space with $\zeta=0.2$, and $\bar{n}=0.1$ for several different κt. It is easy to see that the negative region of

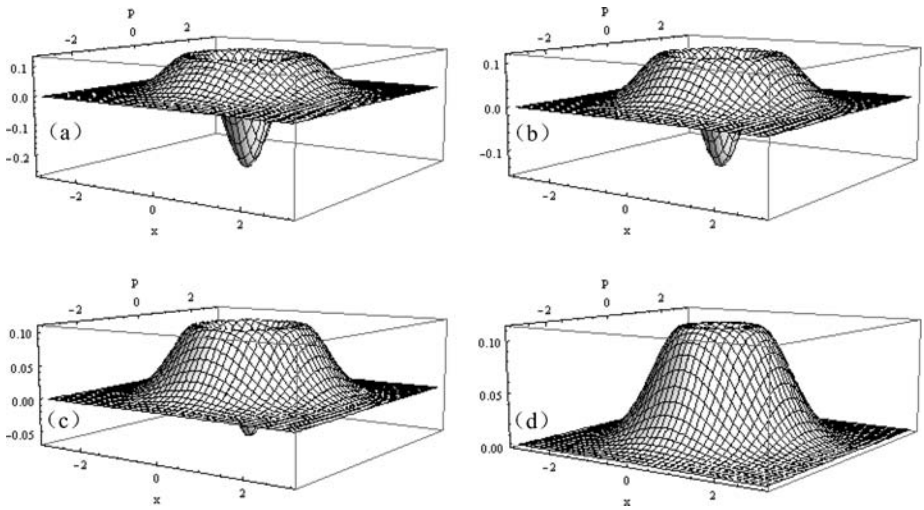

Fig. 3 The evolution of Wigner function of PATICS in thermal channel for different time ($m = 1, q = 0$) (**a**) $\kappa t = 0$, (**b**) $\kappa t = 0.1$, (**c**) $\kappa t = 0.2$, (**d**) $\kappa t = 0.5$

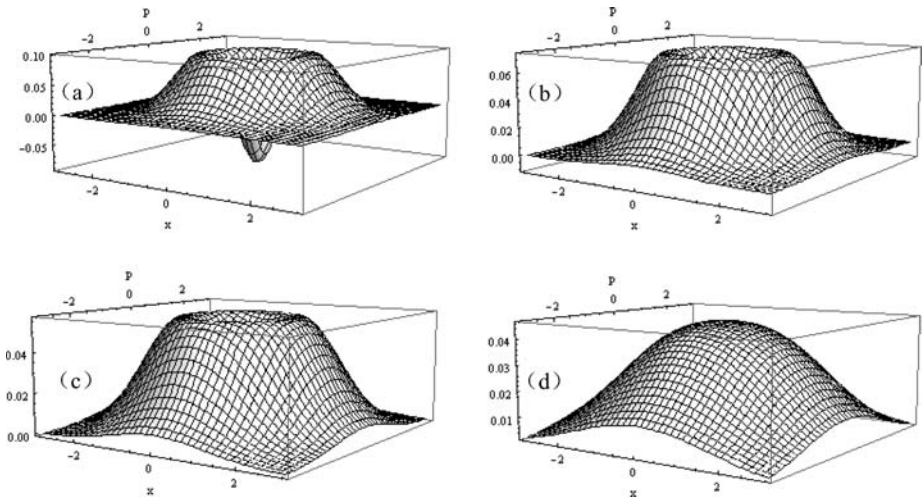

Fig. 4 The evolution of Wigner function of PATICS in thermal channel for different $\bar{n}$ ($m = 1, q = 0$) (**a**) $\bar{n} = 0.5$, (**b**) $\bar{n} = 1.5$, (**c**) $\bar{n} = 3$, (**d**) $\bar{n} = 8$

WF gradually diminishes as the time κt increases. In fact, noticing that $L_n(-|x|^2) > 0$, thus when $(2\bar{n}+1)T - e^{-2\kappa t} > 0$ leading to the following condition

$$\kappa t > \kappa t_c \equiv \frac{1}{2} \ln \frac{2\bar{n}+2}{2\bar{n}+1}, \tag{28}$$

which is independent of the values of ζ and q, there is no negative region for WF in the whole phase space when κt exceeds the threshold value κt_c. The WF gradually becomes Gaussian-type with the time evolution. In Fig. 4, we plot the variation of WF in phase space for different $\bar{n}$ ($m = 1$, and $q = 0$). It is found that the partial negativity of WF decreases gradually as $\bar{n}$ increases for a given time $\kappa t = 0.1$.

5 Conclusions

In this paper, we have introduced the poton-added thermo invariant coherent state. On the basis of the coherent state presentation of Wigner operator, we have discussed its nonclassicality in terms of the negativity of Wigner function after deriving its analytical expression, which is related to Lagurre-Gaussian function. It is shown that when the photon-added number (m) is taken different values, the PATICS exhibits different nonclassicality. Moreover, the nonclassicality is more evident when $m + q$ is an odd number for the PATICS, which indicates that the negativity of WF depend on not only the energy q of the whole system in thermal equilibrium, but also on the photon-added number m. Then we have investigated its decoherence in thermal environments in terms of the partial negativity of WF. It is found that the negative region of WF gradually diminishes as the time κt increases and that there is no chance for WF to present the negative region when the decay time $\kappa t > \frac{1}{2}\ln\frac{2\bar{n}+2}{2\bar{n}+1}$.

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Appendix: Derivation of (11)

Using the coherent state representation (9) of Wigner operator $\Delta(\alpha, \alpha^*)$, and noticing the overlap between coherent state $|\beta\rangle$ and number state $|n\rangle$, i.e.,

$$\langle\beta|n\rangle = \langle 0|\exp\left[-\frac{1}{2}|\beta|^2 + \beta^* a\right]|n\rangle = \frac{1}{n!}\beta^{*n}e^{-\frac{1}{2}|\beta|^2}\langle 0|a^n|n\rangle = \frac{1}{\sqrt{n!}}\beta^{*n}e^{-\frac{1}{2}|\beta|^2}, \tag{A.1}$$

we have

$$\begin{aligned}\langle m|\Delta(\alpha, \alpha^*)|n\rangle &= e^{2|\alpha|^2}\int\frac{d^2\beta}{\pi^2}\langle m|\beta\rangle\langle-\beta|n\rangle e^{-2(\beta\alpha^*-\beta^*\alpha)}\\ &= \frac{(-1)^n e^{2|\alpha|^2}}{\sqrt{m!n!}}\int\frac{d^2\beta}{\pi^2}\beta^m(\beta^*)^n e^{-|\beta|^2-2\alpha^*\beta+2\alpha\beta^*},\end{aligned} \tag{A.2}$$

then using the integration formula

$$H_{m,n}(\xi, \eta) = (-1)^n e^{\xi\eta}\int\frac{d^2z}{\pi}z^n z^{*m}e^{-|z|^2+\xi z-\eta z^*}, \tag{A.3}$$

we have

$$\langle m|\Delta(\alpha)|n\rangle = \frac{(-1)^{n+m}e^{-2|\alpha|^2}}{\sqrt{m!n!}}H_{m,n}(-2\alpha, -2\alpha^*) = \frac{e^{-2|\alpha|^2}}{\sqrt{m!n!}}H_{m,n}(2\alpha, 2\alpha^*), \tag{A.4}$$

which is just (11).

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